UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 26th APRIL 2012

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

The Actuarial Profession
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SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **E** The smallest four-digit positive integer is 1000. Each of the subsequent integers up to and including 1022 has at least two digits the same. However, all digits of 1023 are different so this is the required integer.

2. **C** $1.01 \div 2 = 1 \div 2 + 0.01 \div 2 = 0.5 + 0.005 = 0.505$.

3. **E** An integer will have exactly one factor other than 1 and itself if, and only if, it is the square of a prime. Of the options given, the only such number is 25. Its factors are 1, 5, 25.

4. **C** None of the letters J, N, R has an axis of symmetry, so these letters cannot look the same when reflected in a mirror, no matter how the mirror is held. However, the letters U, I, O all have at least one axis of symmetry, so each may look the same when reflected in a mirror.

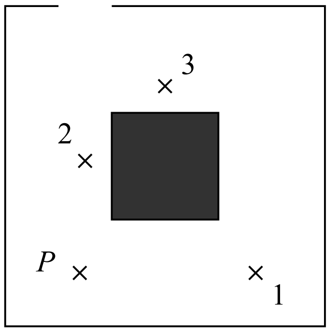
5. **D** $2012 - 1850 = 162$.

6. **D** The first two views of the cube show that I, M, U, O are not opposite K. So P is opposite K. Similarly, the second and third views show that I is opposite O. So the remaining two faces, M and U, must be opposite each other.

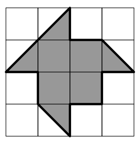
7. **B** Two medium cartridges can print as many pages as three small cartridges, i.e. 1800 pages. So three medium cartridges can print $1800 \times 3/2$ pages, i.e. 2700 pages. This is the same number of pages as two large cartridges can print, so one large cartridge can print $2700 \div 2$, i.e. 1350, pages.

8. **B** The 480 ml in Tommy's tankard represents three quarters of its capacity. So, one quarter of the capacity must be $480 \text{ ml} \div 3 = 160 \text{ ml}$.

9. **C** The person at position P can see exactly two of the other three people, so this person is Caz. The people he can see are at positions 1 and 2 and are Bea and Dan, each of whom can see exactly one person − Caz. This leaves Ali at position 3 − a position from which none of the three people can be seen, so all of the information given is consistent with Caz being at P.

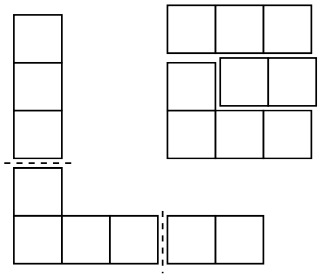


10. **E** The diagram shows the region of overlap, which has area 6 cm^2.



11. **B** We concentrate initially on the units digits of the numbers given, noting that the 3 comes first, so is positive. Now $3 + 7 = 10$ but there is no way to combine 5 and 9 to get a units digit 0. So we must use $3 - 7$. Hence, in the calculation, 67 must be preceded by a minus sign. Now $123 - 67 = 56$. So we need to get an extra 44 by combining 45 and 89. The only way to do this is $89 - 45$. So the correct calculation is $123 - 45 - 67 + 89$. It has two minus signs and one plus sign, so $p - m = 1 - 2 = -1$.

12. **B** None of the pieces which Laura uses to make the 3×3 square can be more than 3 units long. Both the horizontal and vertical portions of the original shape are longer than 3 units, so at least two cuts will be required. Hence Laura will need at least three pieces and the diagrams on the right show that the task is possible using exactly three pieces.



13. **C** Note that p is a factor of both 15 and 18. So p is either 1 or 3. If $p = 1$ then $w = 15$. However, if $w = 15$ then r is not an integer.
So $p = 3, w = 5, x = 6$. The values of the other input factors may now be calculated: $r = 8, s = 10, v = 2, z = 7, q = 5, y = 4, t = 6$.
So A + B + C + D + E = 6 + 25 + 48 + 40 + 42 = 161.

$\times$	p	q	r	s	t
v	A	10		20	
w	15	B	40		
x	18		C	60	
y		20		D	24
z			56		E

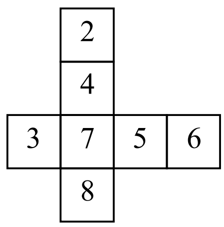
14. **A** Note that 96 is a multiple of 6, so the 97th symbol is the same as the first, the 98th symbol is the same as the second and the 100th and 101st symbols are the same as the fourth and fifth symbols respectively.

15. **E** In total, the fraction of tulips which are either yellow or red is $\frac{1}{2} + \frac{1}{3} \times \frac{1}{2} = \frac{2}{3}$. So one third of the tulips are pink or white. Of these, one quarter are pink, so the fraction of tulips which are white is $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$.

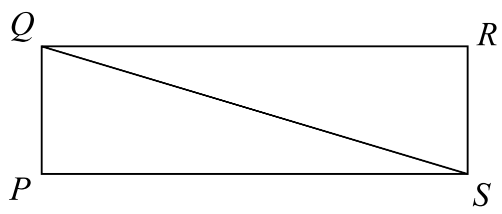
16. **D** Normally, Beth reads pages 1, 4, 7, ... ; Carolyn reads pages 2, 5, 8, ... ; George reads pages 3, 6, 9, ... When Beth is away, Carolyn reads all the odd-numbered pages, whilst George reads all the even-numbered pages. So the pages which are read by the person who normally reads that page are numbered 5, 11, 17 (Carolyn) and 6, 12, 18 (George).

17. **A** The number of boys in the class is $(24 - 6) \div 2 = 9$. So there are 9 boys and 15 girls.
Hence the required ratio is 5 : 3.

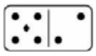
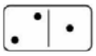
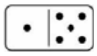
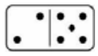
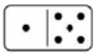
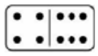
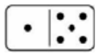
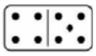
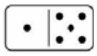
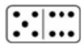
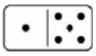
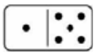
18. **D** Note that the number which replaces x appears in both the row and the column. Adding the numbers in the row and the column gives $2 + 3 + 4 + 5 + 6 + 7 + 8 + x = 2 \times 21 = 42$. So $35 + x = 42$ and hence $x = 7$. The diagram shows one way in which the task may be accomplished.

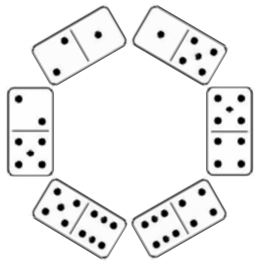


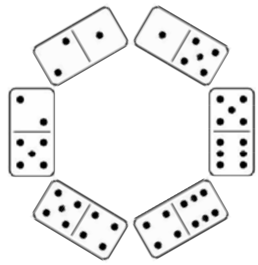
19. **E** As $\angle QPS = 90°$, $\angle PSQ + \angle PQS = 90°$.
So, since the ratio of these angles is 1:5, $\angle PSQ = 15°$ and $\angle PQS = 75°$.
Now $\angle QSR = \angle PQS$ (alternate angles).
So $\angle QSR = 75°$.



20. **A** 50 months = 4 years and 2 months; 50 weeks and 50 days = 57 weeks and 1 day, i.e. just over 1 year and 1 month. So Aroon is just over 55 years and 3 months old and will, therefore, be 56 on his next birthday.

21. **B** Exactly two dominoes have a ‘1’ and exactly two dominoes have a ‘2’ so the dominoes    must be arranged as shown. So  cannot be adjacent to . Clearly,  cannot be adjacent to  either, but it is possible to form a ring with  adjacent to  or with  adjacent to . These are shown below. So only two of the dominoes cannot be placed adjacent to .





22. **B** The original hexagon has been divided into seven regular hexagons and twelve equilateral triangles. Six equilateral triangles are equal in area to one smaller hexagon, so the large hexagon is equal in area to nine of the smaller hexagons. (This may also be deduced from the fact that their sides are in the ratio 3:1.) The shaded area consists of one smaller hexagon and six equilateral triangles, which is equivalent to the area of two of the smaller hexagons. So $\frac{2}{9}$ of the large hexagon is shaded.

23. **A** If either of the first two digits of the number is changed, the units digit will still be 0. Therefore the new number will be either 000 or a non-zero multiple of 10 and so will not be prime. If the units digit is changed then the possible outcomes are 201, 202, 203, 204, 205, 206, 207, 208, 209. The even numbers are not prime and neither are 201 (3×67), 203 (7×29), 205 (5×41), 207 (3×69), 209 (11×19).
So none of the numbers on Peter's list is prime.

24. **D** After 500 games, I have won $500 \times \frac{49}{100} = 245$ games. So I have lost 255 games. Therefore I need to win the next 10 games to have a 50% success rate.

25. **A** The sum of the interior angles of a triangle is 180°.
Therefore $5x + 3y + 3x + 20 + 10y + 30 = 180$, i.e. $8x + 13y = 130$.
As x and y are both positive integers, it may be deduced that x is a multiple of 13.
Also, since $y \geqslant 1$, $x \leqslant \dfrac{117}{8}$ so the only possible value of x is 13. If $x = 13$ then $y = 2$, so $x + y = 15$.